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MODELS OF CONFLICT,
WITH EXPLICIT REPRESENTATION
OF COMMAND AND CONTROL
CAPABILITIES AND VULNERABILITIES

by

Donald P. Gaver

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This report describes combat models between two forces in which command and control facilities actively enhance combat effectiveness, but are also vulnerable. Numerical examples are given; a computer programs listing is included.

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EXECUTIVE SUMMARY

This report describes dynamic combat models that reflect the effect of information flows together with attrition capability upon combat progress and outcome. Command and Control assets for each participant are modeled as endowed with the capacity to guide combat; C^2 is also vulnerable in that it may be deliberately targetted and reduced in effectiveness. Physical attrition is modeled first by a deterministic rate process (Lanchesterian in nature), secondly by a stochastic process related to the first by ideas related to those of stochastic difference and differential equations.

The models are best exercised and explored on an interactive computer display. A FORTRAN program exists for this purpose, with displays of hypothetical "historical" combat outcomes now appearing in tabular form. Graphical displays will be provided in future work.

It seems likely that this model simulation can be developed into a gaming tool, very conveniently playable by two persons who can elect various strategies for force allocation, play the game, and learn from the results. An elaboration of the model may serve as a means for assessing the importance of increased effectiveness of equipment, either with respect to firing rate and accuracy or time for information flow.

MODELS OF CONFLICT, WITH EXPLICIT REPRESENTATION OF COMMAND AND CONTROL CAPABILITIES AND VULNERABILITIES

Donald P. Gaver

1. INTRODUCTION

Many, if not most, conflicts between opposing forces R and B are conducted under some form of Command, Control and Communications (C^3) establishment supervision. Yet few, if any, simple analytical models seem to attempt portrayal of the relationship between C^3 and combat effectiveness. This report suggests models involving C^3 capabilities and vulnerabilities, and indicates the manner in which the models suggested may be utilized in a gaming context.

Models constructed in the present manner were proposed by Gaver and Tonguç (1979). In that study the opposing forces were each split into two groups characterized by their respective information states: those "in the know," or capable of engaging in efficient attrition activities, and those "in the dark," and capable only of less appropriate action, or none at all. It was illustrated by Tonguç (1980) that a capability for quick transition from one information state to another could sometimes outbalance raw physical capability, such as firing rate and single-shot kill probability, thus acting as a "force multiplier."

Once the potential of the C^3 component is recognized, the latter also becomes a potential target. It is, therefore, of interest to incorporate the C^3 component explicitly into attrition-type models, and then to exercise the models so as to expose vulnerabilities and possibly suggest sensible doctrine. We make a stab at this program here in a highly simplified manner, feeling that informative elaborations may well be suggested after an initial look. The idea is to stray in a gingerly fashion into the area between classical Lanchester combat theory and the extensive and elaborate terrain of the modern wargame.

2. MODELS OF POSITION DEFENSE

2.1. Model I: Defense of a Stronghold (or Bastion or Beachhead)

An initial model for this situation was presented by Gaver and Tonguç (1979); see Tonguç (1979); henceforth call this the GT Model. Suppose an R-force of size R attacks a bastion (e.g. beachhead or defended position) held by B. The significance is that B is advantageously located, and in GT was assumed to suffer no casualties initially, while R is exposed and vulnerable and can only succeed by (a) surprise, or (b) B's inability to critically diminish R before being overrun. Suppose that R's speed of advance is (nearly) constant, and that R wins if $R(t_0) \geq kB$, where $R(t_0)$ is R's force size when the stronghold is reached at time t_0; k represents R's necessary advantage over B at final stages or "hand-to-hand" in order to win.

In the GT model, (and here) it was (and is) assumed that B's divide into two combat groups: one in number $B_u(t)$, e.g. the number of those able to fire in an ineffective, specifically unaimed, manner, and another of size $B_a(t)$, e.g. the number of those capable of firing in a more effective, specifically aimed way. All sorts of refinements are possible, but for the present two information states are sufficient.

In GT it was assumed that dynamic transition between $B_u(t)$ and $B_a(t)$ occurred: the rate of transition $B_u(t) \rightarrow B_a(t)$ measured the power of the C^3 system. However, no attempt was made to represent that system as an explicit entity, itself being vulnerable and hence an inviting target. In the present model assume that the C^3 assets of B <u>are</u> vulnerable to R, and that the rate of transition from $B_u(t)$ to $B_a(t)$ is made possible by the C^3 force $B_c(t)$; the latter's effectiveness can in turn be effected by R's actions. In other words, R can attempt to, or inadvertently, target $B_c(t)$ -- and only $B_c(t)$ in the present model -- using the force $R_{cc}(t)$ assigned for that purpose. To the extent that

$R_{cc}(t)$, called the Counter-C^3 (C-C^3) Force, is effective, the C^3 capability of B, namely $B_c(t)$, is reduced by temporary suppression or outright destruction. Such reduction in turn adversely affects the quality of B's response to R's attack. Since in this model the effort (in terms of force size) allocated to C-C^3 activity is removed from the Red active list, thus diminishing R_A and hence the number available to encounter B once the bastion is reached, there is clearly a trade-off opportunity for R. Too large an active force R_A at the expense of C-C^3, R_{cc}, allows extensive attrition by B on R_A free of charge: $B_c(t)$ may work at full effectiveness. On the other hand, too large a C-C^3 force obviously penalizes the attacking force, R_A. Similar choices exist for B. Only by setting down a quantitative representation of the combat dynamics and studying its implications numerically can informed intuition be developed that may lead to a wise trade-off.

Lanchester-Style Equations

Here are some specific Lanchester-style differential equations that represent the dynamics described above in words.

- Blue's C^3 Interactions; Red's Counter-C^3.

$$\frac{dB_u(t)}{dt} = - C_{ua}(B_c(t), B_u(t)) \tag{2.1}$$

$$\frac{dB_a(t)}{dt} = C_{ua}(B_c(t), B_u(t)) \tag{2.2}$$

$$\frac{dB_c(t)}{dt} = C_{cc}(B_c(t), R_{cc}(t)) \tag{2.3}$$

These equations represent the rate of change of the B-force segments with time. They describe only C^3-related activities, since physical change (attrition) of B_u and B_a has not been allowed in the present scenario. Expression (2.1) states that the rate of change reduction of the Blue ineffective (unaimed) force depends

through the function $C_{ua}(.\ ,\ .)$ upon the capacity of the Blue C^3 activity, measured by $B_c(t)$, and upon the number of ineffective forces, $B_u(t)$, awaiting conversion to the effective state. Notice that this expression is left general; $C_{ua}(.\ ,\ .)$ can be specified at will, and must represent the general features of the C^3 activity, including sensor performance and output analysis as well as communication. One simple, tentative, but specific representation might be

$$C_{ua}(B_c(t),\ B_u(t)) = \theta_{ua} B_c(t)\ B_u(t), \qquad (2.1,a)$$

θ_{ua} being a positive constant. This expresses the appealing intuition that the rate of transfer of B_u to B_a should increase jointly with the C^3 capability, B_c, and the number available for change, B_u. On the other hand, (2.1,a) does not reflect processing constraints: if each C^3-equivalent B_c - unit can service one B_u - unit at a time, then the appropriate function should resemble

$$C_{ua}(B_c(t),\ B_u(t)) = \theta_{ua} \min(KB_c(t),\ B_u(t)); \qquad (2.1,b)$$

here K represents the conversion factor that allows combat units (e.g. tanks) to be interchanged for sensor-communication units, or "channels" for short. Other forms for the conversion rate C_{ua} may be derived, possibly by modeling this operational segment in the light of empirical study of any data that happens to be available.

Together, the two equations (2.1) and (2.2) simply state that a decrease in $B_u(t)$ translates into an increase in $B_a(t)$ during the time period (t, t+dt). For simplicity, there is no attrition of B by R, except for that allowed to deplete B's C^3 capability, $B_c(t)$.

The equation (2.3) states that the rate of decrease of the Blue C^3 force, B_c, depends upon the magnitude of Red Counter-C^3 activity, $R_{cc}(t)$, as well as

Blue's C^3 activity, $B_c(t)$. Tentatively illustrate by the simple relationship

$$C_{cc}(R_{cc}(t)) = - \phi \cdot R_{cc}(t), \qquad (2.3,a)$$

ϕ being a constant. This is a conventional Lanchester aimed-fire model. Another similar model might better reflect R's lack of knowledge of B_c location:

$$C_{cc}(R_{cc}(t)) = - \phi \cdot \left(\frac{B_c(t)}{B_o} \right) \cdot R_{cc}(t) \qquad (2.3,b)$$

where B_o is proportional to the area in which B_c is concentrated, and on which R_{cc} concentrates activity. This is essentially an unaimed fire model of classical Lanchester vintage. There may be reason to transition from (2.3,b) to (2.3,a) during the course of the engagement, as information about Blue C^3 increases.

- Blue and Red Physical Attrition.

$$\frac{dR_{cc}}{dt} = - \alpha_{cc}(R_{cc}(t), B_u(t), B_a(t)) \qquad (2.4)$$

$$\frac{dR_A}{dt} = - \alpha_A(R_A(t), B_u(t), B_a(t)) \qquad (2.5)$$

The rate functions $\alpha_{cc}(\cdot)$ and $\alpha_A(\cdot)$ represent the physical attrition exacted by R on the two identified B-force components: the C-C^3 force R_{cc}, and the active attacking force R_A. It is the magnitude of $R_A(t_0)$, i.e. the attack force survivorship at the time t_0 when the bastion is reached, that determines whether the bastion is actually taken. However, the size effectiveness and vulnerability of R_{cc} can indirectly but decisively influence the latter variable and hence the conflict outcome. The interplay of the variables described by equations (2.1) - (2.5) quantifies the qualitative system behaviour.

Here are some specific expressions for physical attrition

$$\alpha_{cc}(R_{cc}(t),\ B_u(t),\ B_a(t)) =$$

$$\rho_{u,cc}\cdot\left(\frac{R_{cc}(t)}{R_0}\right)B_u(t) + \rho_{a,cc}\cdot\left[\frac{kR_{cc}(t)}{kR_{cc}(t) + R_A(t)}\right]B_a(t) \qquad (2.4,a$$

The first right-hand-side (rhs) term represents the rate of attrition of R_{cc} by unaimed B fire; the parameter $\rho_{u,cc}$ is composed of both B_u's fire rate and single-shot kill probability. The second rhs term represents the rate of attrition of R_{cc} by aimed B fire; the parameter $\rho_{a,cc}$ summarizes the joint effect of B_a firing rate and kill probability. The ratio $k\ R_{cc}/kR_{cc} + R_A)$ expresses the fraction of the aimed fire that is directed at the R_{cc} force, where k is a parameter that may be adjusted to account for various combat-related effects, for example: the relative hardness or invulnerability of the R_{cc} force segment as compared to the R_A force segment. It also accounts for the relative exposures of the two forces (R_{cc} and R_A) to aimed fire by B_a. Note that if $k = 1$ the probability that a unit of aimed fire will be directed at the R_{cc} element is simply $R_{cc}/(R_{cc} + R_A)$ -- the fraction of the existing R force at time t that is devoted to C-C^3. If $k = 0$, no aimed shots are directed at the C-C^3 element, instead being concentrated on the attackers, R_A, while if $k \to \infty$ B aimed fire is concentrated on R_{cc}, the C-C^3 element. In short, the simple parameter k expresses the capabilities, vulnerabilities, and priorities of both B and R. It, or a more elaborately developed counterpart, represents combat decision choice, and would appear to have important influence on the progress of the combat.

The rate of decrease of the attack force may be specifically expressed as follows:

$$\alpha_A(R_A(t), B_u(t), B_a(t) =$$

$$\rho_{uA} \cdot \left[\frac{R_A(t)}{R_0}\right] B_u(t) + \rho_{aA} \cdot \left[\frac{R_A(t)}{k\, R_{cc}(t) + R_A(t)}\right] B_a(t) \qquad (2.5,a)$$

Again the first rhs term represents the attrition rate component resulting from the relatively ineffective (unaimed) B fire. The second rhs component represents the attrition rate from aimed fire by B. The fraction $R_A/(k\, R_{cc} + R_A)$ is simply the complement of that appearing in (2.4,a); it expresses in the simplest way a composition of the relative vulnerabilities of R components and the priorities of B.

There appears to be little hope of obtaining insights directly from the differential equations (2.1) - (2.5). Of course such equations can be solved numerically, as was done for the GT model by Tonguç (1979). But another, somewhat simpler, alternative is to express the functions directly in discrete time, taking the time steps to be of unit size; perhaps 0.25 hr. might be appropriate for a start.

Lanchester-Style Equations in Discrete Time

- Blue's C^3 and Physical Interactions; Red's C-C^3

$$B_u(t + 1) = B_u(t) - C_{ua}(B_c(t), B_u(t)) \qquad (2.6)$$

$$B_a(t + 1) = B_a(t) + C_{ua}(B_c(t), B_u(t)) \qquad (2.7)$$

$$B_c(t + 1) = B_c(t) + C_{cc}(B_c(t), R_{cc}(t)) \qquad (2.8)$$

These are easily seen to be the counterparts of (2.1) - (2.3). They may be solved recursively, starting with the initial conditions specified by $B_u(0)$, $B_a(0)$, and $B_c(0)$. Choice of the initial condition by B constitutes a decision, for if initial force size is B = 100 , then choosing $B_c(0) = 20$ leaving $B_u(0) = 80$ and $B_a(0) = 0$ implies more faith by B in his C^3 capabilities and invulnerability than does the choice $B_c(0) = 50$ leaving $B_u(0) = 50$ with $B_a(0) = 0$. The same considerations hold true for R also.

- Red's Physical Attrition

$$R_{cc}(t+1) = R_{cc}(t) - \alpha_{cc}(R_{cc}(t), B_u(t), B_a(t)) \tag{2.9}$$

$$R_A(t+1) = R_A(t) - \alpha_A(R_A(t), B_u(t), B_a(t) \tag{2.10}$$

Here again one starts with initial conditions $R_{cc}(0)$ and $R_A(0)$ and solves recursively to find the Red attacker force size at t_0, when the defended position is reached. If R has considerable faith in the unit effectiveness of its C-C^3, then presumably $R_{cc}(0)$ is chosen to be relatively small, permitting the majority of its resources to be allocated to the attack force $R_A(0)$.

3. NUMERICAL ILLUSTRATIONS OF MODEL I PERFORMANCE

The present version of Model I is as simple as seems consistent with our attempts to blend elements of Command and Control with combat interactions between forces. Even so five state variables are needed to describe system behavior, and their inter-related evolution in time is sufficiently complex to make a direct mathematical discussion appear unprofitable. As an alternative we have elected to create a computer program that produces the numerical sequences of values assumed by the various forces as combat progresses. The computer program is written in FORTRAN; a listing appears in an Appendix. If combat is terminated at a particular time point, t_0, the relationship of $R_A(t)$ to $B_A(t)$ will be assumed to determine the outcome. The latter relationship is itself influenced by decision variables on each side. Here are some options and constraints for the combatants.

- Initial Red attacker force size, $R(0)$, and its division into attackers, $R_A(0)$; and Counter-C^3 (C-C^3), $R_{cc}(0)$; then

$$R(0) = R_A(0) + R_{cc}(0)$$

- Initial Blue defender force size, $B(0)$, and its disposition into Uneffectives/Undesignated, $B_u(0)$; Effectives, $B_a(0)$; and C^3 Forces, $B_c(0)$;

$$B(0) = B_u(0) + B_a(0) + B_c(0)$$

Often, $B_a(0) \simeq 0$ will be reasonable as an initial condition; this would represent surprise by the Red force.

Note that Blue has a technological constraint that limits its C^3 capacity. Two parameters actually play this role: (i) θ_{ua}, representing the rate per unit time of acquisition and transfer, and thus of converting B_u's into B_a's, and (ii) K, the capacity factor according to which force units ($B_c(t)$ = tank equivalents, say) are made equivalent to C^3 units

(sensor-communication combinations or "channels" for short); see (2.1,b) for the rate expression actually used. Initially imagine Blue to be merely endowed with these parameters, and allow them to remain fixed. It may be reasonable for them to change as combat progresses and sensors are disabled, etc. The current computer program can be straightforwardly altered to reflect such combat related damage or degradation.

Blue is assumed to be in possession of the decision parameter, k, see (2.4,a) and (2.5,a): increases in k directs a greater proportion of B_a's fire at R_{cc} forces, while reduction of k concentrates B_a fire at the Red attacking force, R_A; $k = 0$ means exclusive concentration on R_A. Although the present model explicitly makes k a constant throughout the combat period, no such restriction need be at all permanent: it may perhaps be best for B to switch from $k = \infty$ at early stages of combat -- thus maximally reducing R_{cc}'s interference with initial rapid buildup of the B_a's force -- finally switching to $k = 0$ later on so as to concentrate on decimating the R_A force before the defended position is reached. If for example, $B_c(t)$ is (i) hardly reduced at all after a few periods, or (ii) is almost wiped out, then there would seem to be little reason for B_a to target R_{cc} units any longer. Of course if B_u forces are almost entirely converted to B_a's there would again seem to be little reason to target R_{cc}, for B_c's function has been accomplished -- at least so far as the current engagement is concerned.

Switching betwen two extreme values of k is reminiscent of the "bang-bang" policies of control "optimal," but there is no measure of effectiveness or figure of merit yet specified for B.

Numerical Cases

Here are the parameter values selected for initial exercise of the model

$R_A(0) = 120$, $R_{cc}(0) = 30$

ϕ, (attrition rate of B_c by R_{cc}) , = 1 .

$B_a(0) = 0$, $B_u(0) = 80$, $B_c(0) = 20$

$\rho_{uA} = \rho_{u,cc} = 0$

For simplicity we are assuming that the Unaffective Blues have no combat performance capability, but are merely a pool of assets from which Effectives are created by the information from the central C^3 facility. This assumption can easily be modified if desired.

Note carefully that in all that follows the numerical values for various parameters have been chosen for illustrative purposes only, and need bear no close resemblance to any actual values, which are in fact unknown to us. The purposes of the cases discussed is entirely exploratory. But of course we hope that that the suggestions and implications noted will promote interest in further work, leading to model refinements and the use of more nearly correct parameter values, if such can be agreed upon.

Case 1: $\rho_{a,cc} = 1.0$, $\rho_{aA} = 1.0$ (Blue Attrition rates)

$\phi = 1.0$ (Red Attrition rate on Blue C^3)

	Medium Speed Blue C^3		Rapid Blue C^3	
Target:	Attack	$C\text{-}C^3$	Attack	$C\text{-}C^3$
	k=0.2	k=5	k=0.2	k=5
	θ_{ua}=1	θ_{ua}=1	θ_{ua}=2	θ_{ua}=2
↓t				
1	(∞)	(∞)	∞	∞
2	6.0	6.00	3.0	3.0
3	2.97	3.27	1.20	1.50
4	1.57	2.12	0.21	0.60
5	0.56	1.07	0.00	0.00
6	0.00	.06	↓	↓
7	↓	0.00		
8		↓		

Red/Blue Active Force Ratio as Combat Progresses

(Red $C\text{-}C^3$ Attrition Rate $\phi = 1.0$)

Fig. 3.1

The numbers in the above figure suggest that

- Increasing the speed θ_{ua}, (reducing the response time) of Blue C^3 activity has a profound effect upon the Red/Blue Force Ratio: e.g. at $t = 4$ when $k = 0$ (Blues concentrated on Red Attackers) Reds outnumber Blues by 1.6 to 1 if Blue's C^3 speed is $\theta_{ua} = 1$, while doubling that rate to $\theta_{ua} = 2$ under otherwise the same circumstances cuts the ratio to a Blue-favorable 0.21 to 1, i.e. by a factor of about eight.

- For the parameters considered, concentration by Blue on Red Attack forces pays off more than does concentration on the Red $C\text{-}C^3$: e.g. at $t = 4$ with Rapid Blue C^3 the advantage to Red changes from a force ratio of 1.6 to 2.1 with an increased concentration of fire by Blue on Red's $C\text{-}C^3$.

The comparative advantage of Red to Blue ($R_A(t)/B_a(t)$, yielded by Blue emphasis on Red Attackers ($k = 0.2$) instead of Blue emphasis on Red C-C^3 ($k = 5$) diminishes as Blue C^3 speed θ_{ua} decreases. The reason may be that when the crucial Blue C^3 function is relatively weak or slow, it is profitable to spend more effort n its defense, at least until it has served its assigned purpose of converting B_u's to B_a's .

Case 2: $\rho_{a,cc}(t) = 0.25 + 0.075t = \rho_{aA}(t)$

$\phi = 1.0$

In this case a space (and hence time) varying representation of the attrition rate is introduced: at $t = 0$ when the Red force is far away, attrition rates on it are taken to be low, but they increase steadily with decreasing range (increasing t).

	Medium-Speed Blue C^3		Rapid Blue C^3	
	Attack	C-C^3	Attack	C-C^3
	k=0.2	k=5	k=0.2	k=5
	θ_{ua}=1.0	θ_{ua}=1.0	θ_{ua}=2.0	θ_{ua}=2.0
↓t				
1	∞	∞	∞	∞
2	6.0	6.00	3.00	3.00
3	3.35	3.44	1.58	1.60
4	2.25	2.50	0.88	1.12
5	1.37	1.82	0.23	0.60
6	0.40	0.89	0.00	0.00
7	0.00	0.00	↓	↓

Red/Blue Active Force Ratio as Combat Progresses

(Red C-C^3 Attrition Rate $\phi = 1.0$)

Fig. 3.2

There is little qualitative change in the numbers obtained, as compared to those of Fig. 3.1. The change in the attrition rate function, increasing from a small to a larger value as time goes on, allows a larger number of Reds to survive longer than was true for Case 1. Again it seems profitable for Blue to expend most of its energy on Red Attackers.

Case 3: $\rho_{a,cc} = 0.25$, $\rho_{aA} = 0.25$

$\phi = 1.0$ (Red Attrition Rate vs Blue C^3)

$\begin{cases} \theta_{ua} = 0.5 \text{ (Relatively Slow Blue } C^3) \\ \theta_{ua} = 1.0 \text{ (Relatively Rapid Blue } C^3) \end{cases}$

In order to describe the results the entire computer printout is now presented. Parameter values are shown across the page top; note that KC (computer printout) = k (text); the larger (smaller) this parameter becomes the greater (less) is concentration of Blue fire on Red C-C^3.

- Compare cases for which $\theta_{ua} = 0.5$ to $\theta_{ua} = 1.0$: the force nation R_A/B_a for the slower system ($\theta_{ua} = 0.5$) is about twice that for the faster ($\theta_{ua} = 1.0$) system, no matter which firing strategy is adopted (k = 0.2, 5, or 10).
- The time period during which R_A/B_a is above unity (Reds have advantage), or there is near parity, is about 15 to 16 for $\theta_{ua} = 0.5$ (Relatively Slow Blue C^3); the same period is about 8 to 9 for $\theta_{ua} = 1.00$. After that period the R_A force is quickly wiped out, but there are wide differences in the readiness of the B_a's : for k = KC = 5, 10 the B_a's soon reach their maximum of 80, ($t \approx 12$), while at k = KC = 0.2 the maximum is not reached until $t \approx 42$.

32	30.00	0.00	13.33	66.67	.00	0.00

K	KC	RHOUC	RHOUA	RHOAA	RHOAC	PHI	H	THUA	S
1.000	.200	0.000	0.000	.250	.250	1.000	1.000	1.000	0.000

	RCC(T)	RA(T)	BU(T)	BA(T)	BC(T)	RA/BU+BA	RA/BA
1	30.00	120.00	80.00	0.00	20.00	1.50	-1.00
2	30.00	120.00	60.00	20.00	14.00	1.50	6.00
3	29.76	115.24	46.00	34.00	9.80	1.44	3.39
4	29.34	107.16	36.20	43.80	6.88	1.34	2.45
5	28.78	96.77	29.32	50.68	4.86	1.21	1.91
6	28.06	84.81	24.45	55.55	3.46	1.06	1.53
7	27.20	71.79	20.99	59.01	2.49	.90	1.22
8	26.16	58.08	18.50	61.50	1.81	.73	.94
9	24.89	43.97	16.68	63.32	1.34	.55	.69
10	23.28	29.75	15.34	64.66	1.01	.37	.46
11	21.10	15.78	14.34	65.66	.77	.20	.24
12	17.63	2.82	13.57	66.43	.61	.04	.04
13	8.41	0.00	12.96	67.04	.50	0.00	0.00
14	0.00	0.00	12.46	67.54	.46	0.00	0.00
15	0.00	0.00	12.00	68.00	.46	0.00	0.00
16	0.00	0.00	11.54	68.46	.46	0.00	0.00
17	0.00	0.00	11.08	68.92	.46	0.00	0.00
18	0.00	0.00	10.62	69.38	.46	0.00	0.00
19	0.00	0.00	10.16	69.84	.46	0.00	0.00
20	0.00	0.00	9.70	70.30	.46	0.00	0.00
21	0.00	0.00	9.24	70.76	.46	0.00	0.00
22	0.00	0.00	8.78	71.22	.46	0.00	0.00
23	0.00	0.00	8.32	71.68	.46	0.00	0.00
24	0.00	0.00	7.86	72.14	.46	0.00	0.00
25	0.00	0.00	7.40	72.60	.46	0.00	0.00
26	0.00	0.00	6.94	73.06	.46	0.00	0.00
27	0.00	0.00	6.48	73.52	.46	0.00	0.00
28	0.00	0.00	6.02	73.98	.46	0.00	0.00
29	0.00	0.00	5.56	74.44	.46	0.00	0.00
30	0.00	0.00	5.11	74.89	.46	0.00	0.00
31	0.00	0.00	4.65	75.35	.46	0.00	0.00
32	0.00	0.00	4.19	75.81	.46	0.00	0.00
33	0.00	0.00	3.73	76.27	.46	0.00	0.00
34	0.00	0.00	3.27	76.73	.46	0.00	0.00
35	0.00	0.00	2.81	77.19	.46	0.00	0.00
36	0.00	0.00	2.35	77.65	.46	0.00	0.00
37	0.00	0.00	1.89	78.11	.46	0.00	0.00
38	0.00	0.00	1.43	78.57	.46	0.00	0.00
39	0.00	0.00	.97	79.03	.46	0.00	0.00
40	0.00	0.00	.51	79.49	.46	0.00	0.00
41	0.00	0.00	.05	79.95	.46	0.00	0.00
42	0.00	0.00	0.00	80.00	.46	0.00	0.00
43	0.00	0.00	0.00	80.00	.46	0.00	0.00
44	0.00	0.00	0.00	80.00	.46	0.00	0.00
45	0.00	0.00	0.00	80.00	.46	0.00	0.00
46	0.00	0.00	0.00	80.00	.46	0.00	0.00

Red/Blue Force Changes as Combat Progresses (ϕ=1.0)

(K = KC = 0.2)

Fig. 3.3a

47 0.00 0.00 0.00 80.

K	KC	RHOUC	RHOUA	RHOAA	RHOAC	PHI	H	THUA	S
1.000	5.000	0.000	0.000	.250	.250	1.000	1.000	1.000	0.000

	RCC(T)	RA(T)	BU(T)	BA(T)	BC(T)	RA/BU+BA	RA/BA
1	30.00	120.00	80.00	0.00	20.00	1.50	-1.00
2	30.00	120.00	60.00	20.00	14.00	1.50	6.00
3	27.22	117.78	46.00	34.00	9.80	1.47	3.46
4	22.67	113.83	36.20	43.80	7.13	1.42	2.60
5	17.20	108.35	29.07	50.93	5.52	1.35	2.13
6	11.57	101.25	23.55	56.45	4.57	1.27	1.79
7	6.44	92.27	18.99	61.01	4.04	1.15	1.51
8	2.49	80.96	14.95	65.05	3.78	1.01	1.24
9	.32	66.87	11.17	68.83	3.68	.84	.97
10	0.00	50.06	7.48	72.52	3.67	.63	.69
11	0.00	31.93	3.81	76.19	3.67	.40	.42
12	0.00	12.89	.14	79.86	3.67	.16	.16
13	0.00	0.00	0.00	80.00	3.67	0.00	0.00
14	0.00	0.00	0.00	80.00	3.67	0.00	0.00
15	0.00	0.00	0.00	80.00	3.67	0.00	0.00
16	0.00	0.00	0.00	80.00	3.67	0.00	0.00
17	0.00	0.00	0.00	80.00	3.67	0.00	0.00
18	0.00	0.00	0.00	80.00	3.67	0.00	0.00

(k = KC = 5.0)

Fig. 3.3b

19 0.0

K	KC	RHOUC	RHOUA	RHOAA	RHOAC	PHI	H	THUA	S
1.000	10.000	0.000	0.000	.250	.250	1.000	1.000	1.000	0.000

	RCC(T)	RA(T)	BU(T)	BA(T)	BC(T)	RA/BU+BA	RA/BA
1	30.00	120.00	80.00	0.00	20.00	1.50	1.00
2	30.00	120.00	60.00	20.00	14.00	1.50	6.00
3	26.43	118.57	46.00	34.00	9.80	1.48	3.49
4	20.56	115.94	36.20	43.80	7.21	1.45	2.65
5	13.56	111.99	28.99	51.01	5.73	1.40	2.20
6	6.58	106.22	23.26	56.74	4.95	1.33	1.87
7	1.15	97.46	18.31	61.69	4.63	1.22	1.58
8	0.00	83.67	13.69	66.31	4.57	1.05	1.26
9	0.00	67.09	9.11	70.89	4.57	.84	.95
10	0.00	49.37	4.54	75.46	4.57	.62	.65
11	0.00	30.50	0.00	80.00	4.57	.38	.38
12	0.00	10.50	0.00	80.00	4.57	.13	.13
13	0.00	0.00	0.00	80.00	4.57	0.00	0.00
14	0.00	0.00	0.00	80.00	4.57	0.00	0.00
15	0.00	0.00	0.00	80.00	4.57	0.00	0.00
16	0.00	0.00	0.00	80.00	4.57	0.00	0.00
17	0.00	0.00	0.00	80.00	4.57	0.00	0.00
18	0.00	0.00	0.00	80.00	4.57	0.00	0.00
19	0.00	0.00	0.00	80.00	4.57	0.00	0.00
20	0.00	0.00	0.00	80.00	4.57	0.00	0.00
21	0.00	0.00	0.00	80.00	4.57	0.00	0.00
22	0.00	0.00	0.00	80.00	4.57	0.00	0.00
23	0.00	0.00	0.00	80.00	4.57	0.00	0.00
24	0.00	0.00	0.00	80.00	4.57	0.00	0.00
25	0.00	0.00	0.00	80.00	4.57	0.00	0.00
26	0.00	0.00	0.00	80.00	4.57	0.00	0.00
27	0.00	0.00	0.00	80.00	4.57	0.00	0.00
28	0.00	0.00	0.00	80.00	4.57	0.00	0.00
29	0.00	0.00	0.00	80.00	4.57	0.00	0.00
30	0.00	0.00	0.00	80.00	4.57	0.00	0.00
31	0.00	0.00	0.00	80.00	4.57	0.00	0.00
32	0.00	0.00	0.00	80.00	4.57	0.00	0.00
33	0.00	0.00	0.00	80.00	4.57	0.00	0.00
34	0.00	0.00	0.00	80.00	4.57	0.00	0.00
35	0.00	0.00	0.00	80.00	4.57	0.00	0.00

(k = KC = 10)

Fig. 3.3c

Case 4: $\rho_{a,u} = 0.10$, $\rho_{aA} = 0.1$

$\phi = 0.20$, $\theta_{ua} = 0.25$

	$B_u(0)=90(=B_A(\infty))$, $B_c(0)=10$		$B_u(0)=80(=B_A(\infty))$, $B_c(0)=20$		$B_u(0)=60(=B_A(\infty))$, $B_c(0$	
↓t	k: 0	10	0	10	0	10
1	48.0	48.0	24.0	24.0	12.0	12.0
2	24.7	24.7	12.3	12.4	6.1	6.2
3	16.9	17.0	8.4	8.5	4.2	4.2
4	13.0	13.1	6.4	6.5	3.2	3.3
5	10.6	10.8	5.2	5.4	2.6	2.8
10	5.74	6.05	2.62	2.92	1.54	1.92
15	3.95	4.39	1.57	1.95	0.97	1.47
20	2.91	3.43	0.88	1.22	0.46	0.97
30	1.52	1.96	0.00	0.07	0.00	0.00
40	0.44	0.71	↓	0.00	↓	↓

Red/Blue Active Force Ratio as Combat Progresses

Fig. 3.4

The most striking effect visible in the table is that Blue improves his performance relative to Red by decreasing the Effectives and increasing the allocation of his forces to C^3. A nearly 3-to-1 improvement (for B) of the force ratio at $t = 5$ is apparent. The effect of the parameter k, which dictates the fraction of B energy expended to deplete Red C-C^3, is very small.

4. A STOCHASTIC VERSION OF MODEL I.

The evolutionary equations (2.1) - (2.5), or equivalently (2.6) - (2.10) are entirely deterministic, a feature that seems unrealistic since in reality uncertainty and random variability abound. There are several ways in which uncertainty may be allowed to intrude into the formulations; for instance

(i) Through the necessity of estimating parameters ($\rho_{u,cc}$, $\rho_{a,cc}$, k, etc.) from data, assuming the model specification is "correct", or at least adequate.

(ii) Through the necessity of using simplistic models such as (2.1) - (2.5) to represent a more complex reality.

(iii) By explicitly permitting randomness to enter the dynamic equations as an additional driving force.

(iv) Other possibilities; combinations of the above, for example.

Begin by adding a random perturbation term to the discrete time equations (2.6) - (2.10) as in (iii) above:

$$
\begin{aligned}
\underset{\sim}{B}_u(t+1) &= \underset{\sim}{B}_u(t) - C_{ua}\left(\underset{\sim}{B}_c(t), \underset{\sim}{B}_u(t)\right) + \sigma_u\left(\Delta\underset{\sim}{W}_{t+1}(u)\right) \\
\underset{\sim}{B}_a(t+1) &= \underset{\sim}{B}_a(t) + C_{ua}\left(\underset{\sim}{B}_c(t), \underset{\sim}{B}_u(t)\right) + \sigma_a\left(\Delta\underset{\sim}{W}_{t+1}(a)\right) \\
\underset{\sim}{B}_c(t+1) &= \underset{\sim}{B}_c(t) - C_{cc}\left(\underset{\sim}{B}_c(t), R_{cc}(t) + \sigma_c\, \Delta\underset{\sim}{W}_{t+1}(c)\right) \\
\underset{\sim}{R}_{cc}(t+1) &= \underset{\sim}{R}_{cc}(t) - \alpha_{cc}\left(\underset{\sim}{R}_{cc}(t), \underset{\sim}{B}_u(t), \underset{\sim}{B}_a(t)\right) + \sigma_{cc}\left(\Delta\underset{\sim}{W}_{t+1}(cc)\right) \\
\underset{\sim}{R}_A(t+1) &= \underset{\sim}{R}_A(t) - \alpha_A\left(\underset{\sim}{R}_A(t), \underset{\sim}{B}_u(t), \underset{\sim}{B}_a(t)\right) + \sigma_A\left(\Delta\underset{\sim}{W}_{t+1}(A)\right)
\end{aligned}
\qquad (4.1)
$$

The notation $\underset{\sim}{X}$ denotes a random function. The vector of random components $\underset{\sim}{W}_t(u)$, $\underset{\sim}{W}_t(a)$, $\underset{\sim}{W}_t(c)$, $\underset{\sim}{W}_t(cc)$, $\underset{\sim}{W}_t(A)$ will be taken to be one of not necessarily independent Wiener processes, sampled at time points $t = 1, 2, 3, \ldots$. Thus the equations (4.1) turn out to be analogous to Ito-type stochastic differential equations; such equations have been used by Lehoczky and Perla (1978) to describe combat situations.

4.1. Explicit Representation of Random Terms.

By arguments analogous to those of Lehoczky and Perla (1978), or of Gaver and Lehoczky (1977) in a different context, we write down expressions for the scales $\sigma(\cdot)$ of the random components written as $\sigma\cdot\left(\Delta W_{t+1}(\cdot)\right)$ in (4.1). We argue heuristically that if the stochastic processes are nearly Markovian and further are superpositions of many point (e.g. birth-death) processes describing the changes of individuals states in a relatively short time period, then it is reasonable that $\sigma(\cdot)$ be equal to the square-root of the individual drift (deterministic) terms, the latter being given by the expression $C_{ua}\left(\underset{\sim}{B}_c(t), \underset{\sim}{B}_u(t)\right)$, $C_{cc}\left(\underset{\sim}{R}_{cc}(t), \underset{\sim}{B}_u(t), \underset{\sim}{B}_a(t)\right)$, etc. appearing on the rhs of (4.1). The latter is suggested by the approximately Poisson nature of the changes in state of the system over relatively short time intervals. Recall that the standard deviation of a Poisson random variable (state change, here) is equal to the square-root of the mean (state change), or drift in diffusion theory jargon. Furthermore, the random elements $\Delta W_{t+1}(n)$, $\Delta W_{t+1}(a)$, etc., are realizations of independent unit normal or Gaussian random variables. Some readers will recognize the resulting system to be a discrete-time version of Itô-type stochastic differential equations; see Arnold ( ) for basic information.

Here are some explicit examples, following (2.1,b), (2.3), (2.4,a), (2.5,a).

$$\sigma_u\cdot\left(\Delta W_{t+1}(u)\right) = \left[\sqrt{\theta_{ua}\min\{KB_c(t),\ B_u(t)\}}\right]\left(\Delta W_{t+1}(u)\right)$$

$$\sigma_a\cdot\left(\Delta W_{t+1}(a)\right) = \left[\sqrt{\theta_{ua}\min\{KB_c(t),\ B_u(t)\}}\right]\left(\Delta W_{t+1}(a)\right)\ , \qquad (4.2)$$

where

$\Delta W_{t+1}(a) \sim N(0,1)$, i.e. has the Normal distribution with mean 0, and standard deviation 1,

and

$$\Delta W_{t+1}(a) = -\ \Delta W_{t+1}(u),$$

the latter because a fluctuation away from the mean change in one direction--down, say--for $B_u(t)$ is exactly matched by one in the up direction for $B_a(t)$ during time interval t to t+1. Next, using (2.3,b),

$$\sigma_c \cdot \left(\Delta W_{t+1}(c)\right) = \left[\sqrt{\phi \cdot \left(\frac{B_c(t)}{B_o}\right) R_{cc}(t)}\right]\left(\Delta W_{t+1}(c)\right) \tag{4.3}$$

where $\Delta W_{t+1}(c) \sim N(0,1)$, and $\Delta W_{t+1}(c)$ is independent of $\Delta W_{t+1}(a)$ as well as of past values $\Delta W_\tau(c)$, $\tau = 0,1,2,\ldots,t$.

Next,

$$\begin{aligned}\sigma_{cc} \cdot \left(\Delta W_{t+1}(cc)\right) = &\left[\sqrt{\rho_{u,cc} \cdot \left[\frac{R_{cc}}{R_o}\right] B_u(t)}\right]\left(\Delta W_{t+1}(u,cc)\right) \\ &+ \left[\sqrt{\rho_{a,cc}\left[\frac{kR_{cc}(t)}{kR_{cc}(t) + R_A(t)}\right] B_a(t)}\right]\left(\Delta W_{t+1}(a,cc)\right)\end{aligned} \tag{4.4}$$

where $\Delta W_{t+1}(cc,u)$ and $\Delta W_{t+1}(cc,a)$ are independent random elements, both $\sim N(0,1)$, that represent respectively the fluctuation away from mean attrition on the Red C-C[3] facility caused by unaimed (ineffective) fire and by aimed (effective) fire.

Finally, use of (2.5a) provides

$$\begin{aligned}\sigma_A \cdot \left(\Delta \underset{\sim}{W}_{t+1}(A)\right) = &\left[\sqrt{\rho_{uA}\left[\frac{R_A(t)}{R_o}\right] B_u(t)}\right]\left(\Delta \underset{\sim}{W}_{t+1}(u,A)\right) \\ &+ \left[\sqrt{\rho_{aA} \cdot \left[\frac{R_A(t)}{kR_{cc}(t) + R_A(t)}\right] B_a(t)}\right]\left(\Delta \underset{\sim}{W}_{t+1}(a,A)\right).\end{aligned} \tag{4.5}$$

Again, $\Delta W_{t+1}(u,A)$ and $\Delta W_{t+1}(a,A)$ are independent and normally distributed, representing fluctuations around mean attrition on the R_A - component. Under the

present simple circumstances in which approximate independence may be justified, we can actually combine the two "noise" terms in (3.4) and (3.5) to obtain the simpler forms

$$\sigma_{cc} \cdot \Delta W_{t+1}(cc) =$$

$$= \left[\sqrt{\rho_{u,cc}\left[\frac{R_{cc}(t)}{R_o}\right] B_u(t) + \rho_{a,cc}\left[\frac{kR_{cc}(t)}{kR_{cc}(t) + R_A(t)}\right] B_a(t)} \right] \left(\Delta W_{t+1}(cc) \right) \qquad (4.4,a$$

and

$$\sigma_A \cdot \Delta W_{t+1}(A) =$$

$$\left[\sqrt{\rho_{uA}\left[\frac{R_A(t)}{R_o}\right] B_u(t) + \rho_{aA}\left[\frac{R_A(t)}{kR_{cc}(t) + R_A(t)}\right] B_a(t)} \right] \left(\Delta W_{t+1}(A) \right) \qquad (4.5,b$$

This will reduce simulation difficulties by making it unnecessary to sample two normal random variables for each of the last equations. The introduction of correlation is easy, if justified.

It might be pointed out that considerable freedom exists in the choice of the distribution of all of the noise terms $\Delta W_{t+1}(\cdot)$: they need not be normal, nor need they be independent, nor, in fact, need they be independent of the corresponding state value. Of course a random fluctuation that sends, say, $R_A(17)$ formally negative must in truth merely wipe out the A-force, i.e. reduce it to zero. Intuition indicates that a formal (and forbidden) passage below zero for $R_A(t+1)$, and that the latter is unlikely to be extensive by virtue of the small noise variation near $R_A = 0$ for the <u>particular</u> form of (4.5,b). However, choice of a more gaudily variable noise increment, e.g. with $\Delta W_{t+1}(\cdot)$ now chosen to be long tailed, perhaps in a Cauchy-like manner, will likely require more extensive fixing at the boundary. This is no reason to avoid such models, for there is nothing holy about the normal (or Wiener-like) variation save for its appearance

as the noise when a basic Markov structure is assumed to underlie the present models. Actually, over-variation (from the Gaussian/normal) and serial dependence may well usefully represent mixtures of normals (or other) distributions resulting from factors such as terrain, visibility, and many other features which combine to generate departures from the systematic deterministic models analogous to the classical Lanchester forms (2.1) - (2.5).

5. MODEL II: TWO FORCES IN COMBAT WITH MUTUAL ATTRITION

The model described earlier dealt with a special situation in which one force (Red) was the attacker of another in a defended position (Blue). Lack of symmetry was evident. The model of this section represents an equally stylized but now symmetric situation in which both forces are capable of causing attrition on each other. Once again, guidance is furnished by explicitly represented, and vulnerable C^3 agencies. Our models allow for different targetting strategies.

Notation is as follows:

$R_a(t)$ = the number of active Reds at t, meaning the number of R's actually targetted on, and firing at, Blue units.

$B_a(t)$ = the corresponding Blue force size.

$R_i(t)$ = the number of ineffective (or inactive) R's at t, meaning the number of R's currently untargetted and awaiting new assignment.

$B_i(t)$ = the corresponding Blue force size.

$R_c(t)$ = the size of the Red C^3 agency at t, i.e. the force responsible for switching R_i - units to R_a - units.

$B_c(t)$ = the corresponding Blue force size.

The quantities $R_c(t)$ and $B_c(t)$ are to be viewed as the command and control authorities responsible for the individual direct combat elements on their respecti sides. The state variables $R_a(t)$, $R_i(t)$, $B_a(t)$, $B_i(t)$ may be thought of in units of individual tanks or ships, or as aggregations such as battalions, companie naval task groups, or whatever is appropriate for the particular situation under consideration. The intention of the present model is to simply express changes in the respective force sizes in terms of rate processes--both information transfer rates and physical attrition rates--and in terms of initial allocations of resource

It is clear that when Red and Blue forces come into contact a variety of possible behaviors may occur. The models developed here are intended to represent the

consequences of a few simplified versions of the true complexity possible. In particular, they permit the study of different target category priority schemes.

Red(Blue) Force State Equations

The general form of the state change equations now follows. They are given only for Red, but the Blue equations are symmetric.

$$R_a(t+1) = R_a^*(t) - \underbrace{C_{BRA}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t))}_{\text{Physical Attrition, } R_a} + \underbrace{D_{RA}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t), B_i^*(t), B_c^*(t))}_{\text{Effect of Information State Change, } R_a} \quad (5.1)$$

$$R_i(t+1) = R_i^*(t) - \underbrace{C_{BRI}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t))}_{\text{Physical Attrition, } R_i} + \underbrace{D_{RI}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t), B_i^*(t), B_c^*(t))}_{\text{Effect of Information State Change, } R_i} \quad (5.2)$$

$$R_c(t+1) = R_c^*(t) - \underbrace{C_{BRC}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t))}_{\text{Red } C^3 \text{ Capacity Attrition and Suppression}} + \underbrace{D_{RC}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t))}_{\text{Red } C^3 \text{ Capacity Restoration and Recovery}} \quad (5.3)$$

- Additionally, to compute R_a^*, R_i^*, R_c^* from R_a, R_i, R_c, utilize the appropriate physical constraints:

$$R_a^*(t+1) = \min\left[R_a(0) + R_i(0),\ \max\{R_a(t+1),\ 0\}\right]$$

$$R_i^*(t+1) = \min\left[R_a(0) + R_i(0),\ \max\{R_i(t+1),\ 0\}\right]$$

$$R_c^*(t+1) = \min\left[R_c(0),\ \max\{R_c(t+1),\ 0\}\right]$$

- Initial conditions, i.e. values of $R_a(0)$, $R_i(0)$, $R_c(0)$, are required to start the process, after which the various force sizes are calculated recursively in time.
- Abbreviate: C_{BRA}(arguments as in (5.1)) $\equiv C_{BRA}(t)$

  D_{RA}(arguments as in (5.1)) $\equiv D_{RA}(t)$,

  <u>etc.</u>
- Blue State Equations resemble (5.1), (5.2), (5.3) with changes in rate definition and notation

<u>Red</u>	<u>Blue</u>
C_{BRA}	$C_{RBA}(B_a^*(t),\ B_i^*(t),\ B_c^*(t),\ R_a^*(t))$, <u>etc.</u>
D_{RA}	D_{BA}
C_{BRI}	C_{RBI}
D_{RI}	D_{BI}
C_{BRC}	C_{RBC}
D_{BRC}	D_{RBC}

There now follow some specific expression for the above rates, intended to illustrate possible effort allocation strategies.

A. Greedy Allocation (Blue vs. Red)

Priority Order: Red Actives, Red Inactives, Red C^3

a) $C_{BRA}(t) = \rho_A \, B(R_a^*(t))$,

$B(R_a^*(t))$ = Number of Blue Actives vs. Red Actives

$$= \min\left[B_a^*(t), R_a^*(t)\right] \tag{5.4}$$

- Note that all B_a's fire one-on-one on R_a's if B_a force < R_a force; otherwise one-on-one until R_a targets are insufficient, leaving $B_a^*(t) - R_a^*(t)$ to be used against R_i's, which are next on the priority list.

b) $C_{BRI}(t) = \rho_I \, B(R_i^*(t))$,

$B(R_I^*(t))$ = Number of Blue Actives vs. Red Inactives

$$= \min\left[\max\{B_a^*(t) - B(R_a^*(t)), 0\}, R_i^*(t)\right] \tag{5.5}$$

c) $C_{BRC}(t) = \rho_c \, B(R_c^*(t))$

$B(R_c^*(t))$ = Number of Blue Actives vs. Red C^3

$$= \min\left[\max\{B_a(t) - B(R_a^*(t)) - B(R_i^*(t)), 0\}, R_c^*(t)\right] \tag{5.6}$$

- Change of priority is easily accomplished: if priority sequence is

  x first, y second, z third

$$C_{BRx}(t) = \rho_x\, B(R_x^*(t)), \qquad B(R_x^*(t)) = \min\left[B_a^*(t),\, R_x^*(t)\right]$$

$$C_{BRy}(t) = \rho_y\, B(R_y^*(t)), \qquad B(R_y^*(t)) = \min\left[\max\{B_a^*(t) - B(R_x^*(t)),\, 0\},\, R_y^*(t)\right] \tag{5.7}$$

$$C_{BRz}(t) = \rho_z\, B(R_z^*(t)), \qquad B(R_z^*(t)) = \min\left[\max\{B_a^*(t) - B(R_x^*(t)) - B(R_y^*(t)),\, 0\},\, R_z^*(t)\right]$$

- Same, with obvious changes, for Red on Blue
- Attrition rates of Blue, by Red, are denoted respectively by β_a, β_i, β_c.
- Attrition rates can be made time (space) dependent.
- Note that the attrition law is assumed for illustration only to be in accordance with a Lanchesterian linear law. Other types of laws can clearly replace this one.

B. Proportional Allocation (Blue vs. Red)

a) $$C_{BRA}(t) = \rho_A \left[\frac{k_A\, R_a^*(t)}{k_A\, R_a^*(t) + k_I\, R_i^*(t) + k_C\, R_c^*(t)}\right] B_a^*(t) \tag{5.8}$$

- k_A, k_I, k_C are control "constants", representing target priorities. They can be changed or combat progresses, if desired. If $k_A \mid \infty$ (practically $k_A/k_I \simeq k_A/k_C \simeq 10$ should do) there is heavy emphasis on R_a's
- ρ_A is attrition rate (firing rate times single-shot kill probability).

b) $$C_{BRI}(t) = \rho_I \left[\frac{k_I\, R_i^*(t)}{k_A\, R_a^*(t) + k_I\, R_i^*(t) + k_C\, R_c^*(t)}\right] B_a^*(t) \tag{5.9}$$

$$c)\quad C_{BRC}(t) = \rho_c \left[\frac{k_C\, R_c^*(t)}{k_A\, R_a^*(t) + k_I\, R_i^*(t) + k_C\, R_c^*(t)} \right] B_a^*(t) \qquad (5.10)$$

- Same for Red on Blue with appropriate notational changes.
- This is an alternative strategy that allocates some B_a-effort to <u>all</u> components of the Red force. It samples rather than gulps. There are many possible alternatives.

Red(Blue) Information State Change Equations

Here are some sample equations that are tentatively proposed to model the effects of philosophy and capability of the C^2 component.

(1) Tight Central Control(Reds); C^2 Attrition

Only R_a's may fire at Blues. When a Red firing engagement terminates, it returns to the R_i status (becomes inactive). The R_c force acts to change or return R_i's to R_a's. Thus

$$(a)\quad D_{RA}(t) = \underbrace{\min\{C_R\, K_R\, R_C^*(t),\ C_R\, R_i(t),\ R_i(t)\}}_{(R_i \to R_a \text{ rate; } C^2 \text{ process})} - \underbrace{\left[C_{RBA}(t) + C_{RBI}(t) + C_{RBC}(t)\right]}_{\begin{array}{l}\text{(Number of Red } \underline{\text{vs}} \text{ Blue Engagements Terminated} \\ \text{in } t^{th} \text{ Period, or Number of } R_a\text{'s Released to} \\ R_i.)\end{array}} \qquad (5.11)$$

$$(b)\quad D_{RI}(t) = -\, D_{RA}(t) \qquad (5.12)$$

(c) $D_{RC}(t) = 0$. (No Red C^2 recovery from suppression.) (5.13)

- Note that in this model both physical attrition (which is permanent) and lack of information (temporary, unless C^2 is inadequate) reduce Red effectiveness.

(2) Loose Central Control; C^2 Attrition

Same as (1), but retain only the first term

(a) $D_{RA}(t) = \min\{C_R K_R R_c^*(t), C_R R_i(t), R_i(t)\}$ (5.14)

(b) $D_{RI}(t) = - D_{RA}(t)$ (5.15)

(c) $D_{RC}(t) = 0$ (5.16)

- Note that this model assumes that once an R_i changes to R_a status it remains so until physically removed.
- A model that lies between (1) and (2) may be attractive, in which case one might multiply the second term in (5.11) by a time (distance) dependent factor less than unity to represent the fraction of the terminating Red engagements that must have recourse to central C^2 re-direction.

6. NUMERICAL ILLUSTRATIONS OF MODEL II PERFORMANCE.

In this section we present some examples of Model II behavior. Once again these numerical results are obtained from a computer program written in FORTRAN; a listing appears in an Appendix. The intention is to provide some feeling for the sensitivity of the model to changes in parameters and combat philosophy or priorities. Careful study of the outcomes will be helpful in leading one to comprehend the complex dynamics of this model and others evolving from it.

Discussion. For convenience, consider the columns in the Figure labelled $1 \rightarrow 9$ from the left; e.g. Col. 3 is headed $c_R=1,(2)$, under $\rho_c=0.1$, meaning that Red's C^2 rate is unity, the attrition rate of Blue on Red is $\rho_c=0.1$, and (1) Strong Central Control is in use by both antagonists.

- Each combatant's Active component first grows, as C^2 changes Inactive to Actives, and then dwindles by attrition. This is to be expected.
- Compute Col. 2 and Col. 3: Strong Central Control tends to overload C^2 to the detriment of Red Actives. See also Col. 6 and Col. 7: same lesson for Blue Actives. A tradeoff would occur if S.C.C. by Red was combined with higher attrition rates for Red than Blue, Blue using L.C.C.
- Compare Col. 2 and Col. 4: increasing (doubling) C^2 processing rate under S.C.C. provides a decided initial improvement for Red. but this disappears later; the number of Inactives becomes small, so there are no resources from which to draw to increase the complement of Actives.

	$R_a(t)$				$B_a(t)$			
	ρ_c=0.1		ρ_c=0.5		ρ_c=0.1		ρ_c=0.5	
t	c_R=1,(1)	c_R=1,(2)	c_R=2,(1)	c_R=2,(2)	c_R=1,(1)	c_R=1,(2)	c_R=2,(1)	c_R=2,(2
0	0	0	0	0	0	0	0	0
1	20	20	40	40	10	10	10	10
2	37	39	75	79	18	19	18	19
3	52	57	90	97	24	27	24	27
4	75	91	83	91	34	41	34	41
6	81	87	81	87	37	47	37	47
7	77	82	77	82	40	52	40	52
8	73	77	73	77	42	57	42	57
9	69	71	69	71	43	61	44	61
10	65	65	65	65	45	65	45	65
15	44	38	44	38	48	64	48	52
20	26	23	26	23	44	48	32	36
25	16	13	16	13	35	38	21	26
Col:								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

Fig. 6.1

$c_B = 1.0, \quad _c = 0.10$

Priorities: Actives - Inactives - C^2

(1): Strong Central Control

(2): Loose Central Control

Discussion. This Fig. 6.2 illustrates the effect of changes in target priority under different philosophies of central control.

- All else being equal, S.C.C., indicated by (1), is a handicap as illustrated by comparison of Col. 2 and 4 for Reds, and corresponding Cols. 6 and 8 for Blues. The penalty for Blue is higher possibly because of its original smaller complement of C^2 assets, and hence its poorer traffic handling capability.
- Suppose Blue reverses firing priority, targetting C^2 first, then R_a, finally R_i. Compare Cols. (3) and (7) to Cols. (2) and (6). Note that initially R_a actually improves when C^2 is first priority because R_a is only lightly diminished and R_c remains temporarily adequate. It appears that this strategy change by B is ineffective because Red's C^2 facilities are ample enough to withstand the attack.
- Suppose Red reverses firing strategy, targetting Blue C^2 first, then B_a, and finally R_i. Adopt loose control, (2), for illustration. Note that the effect on B_a of Reds C^2-first strategy is initially small, but as the combat proceeds B_a force size is considerably reduced.

The last two comparisons suggest that primary attach on the opponent's C^2 force is advantageous when that force is meager (or especially vulnerable), while such an attack is actually counterproductive in case the C^2 force is adequate. Of course this simply suggests that high priority is best placed on attacking the weakest point in the system. In the present example weakness is merely a matter of numbers (C^2 force size), whereas in reality there must be an assessment of a potential target's capability or performance rate. Models of the present type, and their offspring and siblings, should be of use for evaluating different proposed targetting strategies or doctrine.

	$R_a(t)$				$B_a(t)$			
	(A-I-C)	(C-A-I)	(A-I-C)	(A-I-C)	(A-I-C)	(A-I-C)	(A-I-C)	(C-A-I
t	c_R=1,(1)	c_R-1,(1)	c_R=1,(2)	c_R=1,(2)	c_R=1,(1)	c_R=1,(1)	c_R=1,(2)	c_R=1,(2
0	0	0	0	0	0	0	0	0
1	20	20	20	20	10	10	10	10
2	37	38	39	39	18	18	19	19
3	52	53	57	57	24	24	27	26
4	64	64	74	75	30	30	34	32
5	75	72	91	91	34	34	41	36
6	81	77	87	88	37	37	47	39
7	77	79	82	84	40	40	52	41
8	73	80	77	80	72	72	57	42
9	69	79	71	76	43	43	61	43
10	65	76	65	71	45	45	65	43
15	44	55	39	51	48	48	64	38
20	26	33	23	33	33	33	48	30
25	16	19	13	20	21	21	38	22
Col:								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

Fig. 6.2

$c_R = c_B = 1$, $\rho_c = \beta_c = 0.10$

Various Priorities

Cases: Cols. 2 & 6
Cols. 3 & 7
Cols. 4 & 8
Cols. 5 & 9

(1): Strong Central Control
(2): Loose Central Control

PROGRAM LISTING MODEL I
(DETERMINISTIC & STOCHASTIC)

: RWAR FORTRAN A NAVAL POSTGRADUATE SCHOOL

```
      REAL BU(100),BA(100),BC(100),RCC(100),RA(100)                     RWA00010
      REAL KC,K,H,THUA,PHI,BO,RO,RHOUC,RHOAC                            RWA00020
      INTEGER T,TO                                                      RWA00030
      REAL*8  TITLE(3,2)                                                RWA00040
      DATA TITLE/'DETERMIN','ISTIC C3',' MODEL  ','STOCHAST','IC C3 MO',RWA00050
     X   'DEL'/                                                         RWA00060
      DATA IX/123456/, IP/0/                                            RWA00070
.     GAVER COMBAT MODEL                                                RWA00080
      RHOUC=0.                                                          RWA00090
      RHOUA=0.                                                          RWA00100
      BU(1)=30.                                                         RWA00110
      BA(1)=0.                                                          RWA00120
      BC(1)=20.                                                         RWA00130
      RCC(1)=30.                                                        RWA00140
      RA(1)=120.                                                        RWA00150
      H=1.                                                              RWA00160
      TO=61                                                             RWA00170
                                                                        RWA00180
      CONTINUE                                                          RWA00190
      WRITE(6,121)                                                      RWA00200
      READ(5,122) K,KC,RHOAA,RHOAC,S,THUA,PHI,ITYPE                     RWA00210
      IF(K .LT. 0.) STOP                                                RWA00220
                                                                        RWA00230
      WRITE(6,111) K,KC,RHOUC,RHOUA,RHOAA,RHOAC,PHI,H,THUA,S            RWA00240
      WRITE(6,123) (TITLE(I,ITYPE),I=1,3)                               RWA00250
      WRITE(6,100)                                                      RWA00260
      RO=RCC(1) + RA(1)                                                 RWA00270
      BO=BA(1)+BU(1)+BC(1)                                              RWA00280
      BUAO=BA(1)+BU(1)                                                  RWA00290
                                                                        RWA00300
      DO 10 T=1,TO                                                      RWA00310
       R1=-1.                                                           RWA00320
       IF(BU(T)+BA(T) .NE. 0.) R1=RA(T)/( BU(T)+BA(T) )                 RWA00330
       R2=-1.                                                           RWA00340
       IF( BA(T) .NE. 0. ) R2=RA(T)/BA(T)                               RWA00350
       R3=-1.                                                           RWA00360
       IF(RA(1) .NE. 0.) R3=(RA(T)-RA(1))/RA(1)                         RWA00370
       R4=-1.                                                           RWA00380
       IF(T .GT. 1 .AND. RA(T).NE. 0.) R4=(BA(T)-RA(T-1))/RA(T-1)       RWA00390
       IT1=T-1                                                          RWA00400
       WRITE(6,101)IT1,RCC(T),RA(T),BU(T),BA(T),BC(T),R1,R2,R3,R4       RWA00410
                                                                        RWA00420
       CUAN=CUA(BC(T),BU(T),THUA,K)                                     RWA00430
       RCUA=SQRT(CUAN)*GAUSS(IX,IP,ITYPE)                               RWA00440
       BU(T+1)=AMAX1(BU(T)-CUAN-RCUA, 0.)                               RWA00450
       BA(T+1)=AMIN1(BA(T)+CUAN+RCUA, BUAO)                             RWA00460
                                                                        RWA00470
       CC=CCC(RCC(T),PHI,BC(T),BO)                                      RWA00480
       BC(T+1)=AMAX1( BC(T)- CC  + SQRT(CC)*GAUSS(IX,IP,ITYPE) , 0. )   RWA00490
                                                                        RWA00500
       CALL ALCC(RCC(T),BU(T),BA(T),RHOUC,RHOAC,RA(T),RO,KC, AC1,AC2)   RWA00510
       RCC(T+1)=AMAX1( RCC(T)-(AC1+AC2) +                               RWA00520
     X                 SQRT(AC1+AC2)*GAUSS(IX,IP,ITYPE) , 0. )        RWA00530
                                                                        RWA00540
       CALL ALA(RA(T),BU(T),BA(T),RCC(T),RHOUA,KO,RHOAA,KC , AL1,AL2)   RWA00550
```

```
      RA(T+1)=AMAX1( RA(T) - (AL1+AL2) +                                RWA00
     X                SQRT(AL1+AL2)*GAUSS(IX,IP,ITYPE) , 0. )          RWA00
      IF(RA(T+1)-RA(T).LT. .01 .AND. RCC(T+1)-RCC(T).LT. .01 .AND.      RWA00
     X  BC(T+1)-BC(T).LT. .01 .AND. BA(T+1)-BA(T).LT. .01               RWA00
     X    .AND. BU(T+1)-BU(T).LT. .01 ) GO TO 15                       RWA00
   10 CONTINUE                                                          RWA00
   15 GO TO 5                                                           RWA00
  100 FORMAT(/'      RCC(T)     RA(T)     BU(T)     BA(T)     BC(T)',   RWA00
     X    ' RA/BU+BA    RA/BA RA(T)-RA(0)/RA(0) RA(T)-RA(T-1)/RA(T-1)') RWA00
C                                                                       RWA00
  101 FORMAT(I3,7F9.2,F17.2,F22.2)                                      RWA00
  111 FORMAT(/,'      K       KC     RHOUC   RHOUA   RHOAA   RHOAC    PHI',  RWA00
     X    '      H     THUA       S',/10F7.3)                          RWA00
  121 FORMAT('1 ENTER K,KC,RHOAA,RHOAC,S,THUA,PHI,TYPE (F6.0):')        RWA00
  122 FORMAT(7F6.0,I6)                                                  RWA00
  123 FORMAT(//31X,3A8)                                                 RWA00
      END                                                               RWA00
C                                                                       RWA00
      SUBROUTINE ALCC(RCC,BU,BA,RHOUC,RHOAC,RA,RO,KC, ALC1,ALC2)        RWA00
       REAL KC                                                          RWA00
       ALC1= RHOUC*RCC*BU/RO                                            RWA00
       ALC2= 0.                                                         RWA00
       IF(RCC .LT. .00001 .OR. KC .LT. .000001) GO TO 10                RWA00
       ALC2= RHOAC*KC*RCC*BA/(KC*RCC + RA)                              RWA00
   10  RETURN                                                           RWA00
      END                                                               RWA00
C                                                                       RWA00
      SUBROUTINE ALA(RA,BU,BA,RCC,RHOUA,RO,RHOAA,KC, AL1,AL2)           RWA00
       REAL KC                                                          RWA00
       AL1= RHOUA*RA*BU/RO                                              RWA00
       AL2= 0.                                                          RWA00
       IF (RA .LT. .00001) GO TO 10                                     RWA00
       AL2= RHOAA*RA*BA/(KC*RCC+RA)                                     RWA00
   10  RETURN                                                           RWA00
      END                                                               RWA00
C                                                                       RWA00
      FUNCTION CUA(BC,BU,THUA,K)                                        RWA00
       REAL K                                                           RWA00
       CUA=AMIN1(THUA*K*BC,THUA*BU,BU)                                  RWA00
       RETURN                                                           RWA00
      END                                                               RWA00
C                                                                       RWA00
      FUNCTION CCC (RCC,PHI,BC,BU)                                      RWA00
       CCC= PHI*RCC*BC/BU                                               RWA01
       RETURN                                                           RWA01
      END                                                               RWA01
C                                                                       RWA01
      FUNCTION GAUSS(IX,IP,ITYPE)                                       RWA01
      REAL A(1000)                                                      RWA01
      GAUSS=0.                                                          RWA01
      IF(ITYPE .EQ. 1) RETURN                                           RWA01
      IF(IP .GT. 0) GO TO 10                                            RWA01
      CALL UNORM(IX,A,1000,1,0)                                         RWA01
      IP=1000                                                           RWA01
   10 GAUSS =A(IP)                                                      RWA01
```



```
      IP = IP - 1                                                       RWA01
      RETURN                                                            RWA01
      END                                                               RWA01
```

PROGRAM LISTING MODEL II (DETERMINISTIC)

: WACT FORTRAN A NAVAL POSTGRADUATE SCHOOL

```
      REAL R(3,100),B(3,100),DRA,DBA                                    WAC00010
      REAL KR,KB, RO(3),BE(3), CBR(3),CRB(3)                            WAC00020
      INTEGER NAME(3)/'A','I','C'/                                      WAC00030
      INTEGER T,TO,VR(3),VB(3), PR(3),PB(3)                             WAC00040
      REAL*8  TITLE(3)                                                  WAC00050
      DATA TITLE/'PRIORITY',' TARGETT','ED MODEL'/                      WAC00060
      DATA IX/123456/, IP/0/                                            WAC00070
.     GAVER COMBAT MODEL  PRIORITY TARGETTED                            WAC00080
      R(1,1)=0.                                                         WAC00090
      R(2,1)=100.                                                       WAC00100
      R(3,1)=20.                                                        WAC00110
      B(1,1)=0.                                                         WAC00120
      B(2,1)=150.                                                       WAC00130
      B(3,1)=10.                                                        WAC00140
      VR(I),I=1,3  ARE PRIORITIES FOR A,I,C RESPE. SAME FOR VB(I)       WAC00150
      VR(1)=1                                                           WAC00160
      VR(2)=2                                                           WAC00170
      VR(3)=3                                                           WAC00180
      VB(1)=1                                                           WAC00190
      VB(2)=2                                                           WAC00200
      VB(3)=3                                                           WAC00210
      TO=61                                                             WAC00220
                                                                        WAC00230
      CONTINUE                                                          WAC00240
      WRITE(6,121)                                                      WAC00250
      READ(5,122) KR,KB,CR,CB,(RO(I),BE(I),I=1,3)                       WAC00260
      IF(KR .LT. 0.) STOP                                               WAC00270
      DO 6 I=1,3                                                        WAC00280
       PR(VR(I))=I                                                      WAC00290
       PB(VB(I))=I                                                      WAC00300
       CRB(I)=-1.                                                       WAC00310
       CBR(I)=-1.                                                       WAC00320
      CONTINUE                                                          WAC00330
                                                                        WAC00340
      BAIO=B(1,1)+B(2,1)                                                WAC00350
      RAIO=R(1,1)+R(2,1)                                                WAC00360
      DRA=-1.                                                           WAC00370
      DBA=-1.                                                           WAC00380
      WRITE(6,111) KR,KB,CR,CB,(RO(I),BE(I),I=1,3)                      WAC00390
      WRITE(6,127) (NAME(VB(I)),I=1,3),(NAME(VR(I)),I=1,3)              WAC00400
      WRITE(6,123) (TITLE(I),I=1,3)                                     WAC00410
      WRITE(6,100)                                                      WAC00420
                                                                        WAC00430
      DO 10 T=1,TO                                                      WAC00440
       IT1=T-1                                                          WAC00450
       WRITE(6,101) IT1,(R(I,T),I=1,3),(B(I,T),I=1,3),                  WAC00460
     X      (CBR(I),I=1,3),(CRB(I),I=1,3),DRA,DBA                      WAC00470
                                                                        WAC00480
       CRB(1)=AMIN1( B(PB(1),T), R(1,T) )                               WAC00490
       CRB(2)=AMAX1(0., AMIN1(B(PB(2),T),R(1,T)-CRB(1)) )               WAC00500
       CRB(3)=AMAX1(0., AMIN1(B(PB(3),T),R(1,T)-CRB(1)-CRB(2)) )        WAC00510
                                                                        WAC00520
       CBR(1)=AMIN1( R(PR(1),T), B(1,T) )                               WAC00530
       CBR(2)=AMAX1(0., AMIN1(R(PR(2),T),B(1,T)-CBR(1)) )               WAC00540
       CBR(3)=AMAX1(0., AMIN1(R(PR(3),T),B(1,T)-CBR(1)-CBR(2)) )        WAC00550
```

```
C                                                                       WAC00
      DRA=AMIN1( CR*KR*R(3,T),CR*R(2,T),R(2,T) )                        WAC00
      DBA=AMIN1( CB*KB*B(3,T),CB*B(2,T),B(2,T) )                        WAC00
      DO 19 I=1,3                                                       WAC00
       CBR(I)=CBR(I)*RO(PR(I))                                          WAC00
       CRB(I)=CRB(I)*BE(PB(I))                                          WAC00
C      DRA=DRA-CRB(I)                                                   WAC00
C      DBA=DBA-CBR(I)                                                   WAC00
   19 CONTINUE                                                          WAC00
C                                                                       WAC00
      R(1,T+1)=AMIN1(RAIO, AMAX1(0., R(1,T)-CBR(VR(1)) + DRA))          WAC00
C                                                                       WAC00
      R(2,T+1)=AMIN1(RAIO, AMAX1(0., R(2,T)-CBR(VR(2)) - DRA))          WAC00
C                                                                       WAC00
      R(3,T+1)=AMIN1(R(3,1), AMAX1(0., R(3,T)-CBR(VR(3))))              WAC00
C                                                                       WAC00
      B(1,T+1)=AMIN1(BAIO, AMAX1(0., B(1,T)-CRB(VB(1)) + DBA))          WAC00
C                                                                       WAC00
      B(2,T+1)=AMIN1(BAIO, AMAX1(0., B(2,T)-CRB(VB(2)) - DBA))          WAC00
C                                                                       WAC00
      B(3,T+1)=AMIN1(B(3,1), AMAX1(0., B(3,T)-CRB(VB(3))))              WAC00
C                                                                       WAC00
   10 CONTINUE                                                          WAC00
C                                                                       WAC00
      GO TO 5                                                           WAC00
C                                                                       WAC00
  100 FORMAT(/'       RA(T)    RI(T)    RC(T)    BA(T)    BI(T)',       WAC00
     X    '    BC(T)' )                                                WAC00
C                                                                       WAC00
  101 FORMAT(13,6F9.2,3F7.2)                                            WAC00
  111 FORMAT(/,'      KR       KB       CR       CB      RUA      BEA      RUI',WAC00
     X    '      BEI      RUC      BEC',/10F7.3)                        WAC00
  121 FORMAT('1 ENTER KR,KB,CR,CB,RUA,BEA,RUI,BEI,RUC,BEC:')            WAC00
  122 FORMAT(10F6.0)                                                    WAC00
  123 FORMAT(//31X,3A8)                                                 WAC00
  127 FORMAT('  PRIORITIES: R-B: ',3A2,12X,'B-R: ',3A2)                 WAC00
      END                                                               WAC00
```

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